

Mark Scheme (Results)

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Pearson Edexcel International Advanced Level In Further Pure Mathematics F2 (WFM02) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for this paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
- e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. e.g. M0 A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph).

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{\text{ will be used for correct ft}}$
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- ★ The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao), unless shown, for example as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general priniciples)

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where $|pq| = |c|$ leading to $x = ...$
 $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving
$$x^2 + bx + c = 0$$
: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, q \neq 0$, leading to $x = \dots$

Method mark for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Notes	Marks
1 (a)	$y = \ln(5+3x) \Rightarrow \frac{dy}{dx} = \frac{3}{5+3x}$	Correct first derivative	B1
	$y = \ln(5+3x) \Rightarrow \frac{dy}{dx} = \frac{3}{5+3x}$ Correct first derivative $\frac{dy}{dx} = \frac{3}{5+3x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{9}{(5+3x)^2} \Rightarrow \frac{d^3y}{dx^3} = \frac{54}{(5+3x)^3}$		
	M1: Continues the process of differentiating and reaches $\frac{d^3y}{dx^3} = \frac{k}{(5+3x)^3}$ oe		
	Note this may be achieved via the quotient	()	
	A1: Correct simplified third derivative. A	llow e.g. $\frac{54}{(5+3x)^3}$ or $54(5+3x)^{-3}$.	(2)
(b)	$y_0 = \ln 5, y_0' = \frac{3}{5}, y_0'' =$	9 ,,,,, 54	(3)
	$\Rightarrow \ln(5+3x) \approx \ln 5 + \frac{3}{5}x - $	20 120	M1
	Attempts all values at $x = 0$ and applies Maclaurin's theorem. Evidence for attempting the values can be taken from at least 2 terms. The form of the expansion must be correct including the factorials or their values. Note that this is "Hence" and so do not allow other methods e.g. Formula Book.		
	$\ln\left(5+3x\right) \approx \ln 5 + \frac{3}{5}x - \frac{3}{5}$	$\frac{9}{50}x^2 + \frac{9}{125}x^3 + \dots$	A1
	Correct expansion. The "ln(5 -	(+3x) = ' is not required.	(2)
(c)	$\ln(5-3x) \approx \ln 5 - \frac{3}{5}x - \frac{9}{50}x^2 - \frac{9}{125}x^3 + \dots$ Correct expansion even if obtained "from scratch" OR for a correct follow through with signs changed on the coefficients of the odd powers of x only in an expansion of the correct form e.g. a polynomial in ascending powers of x.		B1ft
(4)	(5.2)		(1)
(d)	$\ln \frac{(5+3x)}{(5-3x)} = \ln (5+3x)$ $\ln 5 + \frac{3}{5}x - \frac{9}{50}x^2 + \frac{9}{125}x^3 + \dots - \left(\ln x\right)$ Subtracts their 2 different series to obtain a powers of	$5 - \frac{3}{5}x - \frac{9}{50}x^2 - \frac{9}{125}x^3 + \dots$ at least 2 non-zero terms in ascending	M1
	powers of $= \frac{6}{5}x + \frac{18}{125}x$ Correct terms. Allow e.g. 0+		A1

Allow both marks to score in (d) provided the correct series have been obtained in	
(b) and (c) by any means.	
	(2)
	Total 8

Question Number	Scheme	Notes	Marks
2(a)	$\Rightarrow A =,$	$\frac{A}{1} = \frac{A}{2n-1} + \frac{B}{2n+1} + \frac{C}{2n+3}$ $B =, C =$ of to obtain values for A, B and C	M1
	$\frac{1}{8(2n-1)} - \frac{1}{4(2n+1)} + \frac{1}{8(2n+3)}$ or e.g. $\frac{\frac{1}{8}}{(2n-1)}$ Correct partial fractions	or e.g $\frac{1}{16n-8} - \frac{1}{8n+4} + \frac{1}{16n+24}$	A1
(b)	$\frac{1}{8} \left(\frac{1}{1} - \frac{2}{3} + \frac{1}{5} \right)$ $+ \frac{1}{3} - \frac{2}{5} + \frac{1}{5} + \frac{2}{7} + \frac{1}{5} - \frac{2}{7} + \frac{1}{2n-3} - \frac{1}{2} + \frac{1}{2n-1} - \frac{1}{2}$ Uses the method of differences to fine E.g. 3 rows at the start and the s	1/ /7	M1
_	Identifies the correct non-cance	$= \frac{1}{8} \left(1 - \frac{2}{3} + \frac{1}{3} + \frac{1}{2n+1} - \frac{2}{2n+1} + \frac{1}{2n+3} \right)$ elling terms. May be unsimplified. $-3(2n+3) + 3(2n+1)$	A1
	Attempts to combine terms into one fr constant term and at least 2 different a	$\frac{-3(2n+3)+3(2n+1)}{1)(2n+3)} = \dots$ raction. There must have been at least one algebraic terms with at least 3 terms in the sombining the fractions.	dM1

$=\frac{n(n+2)}{3(2n+1)(2n+3)}$	cao	A1
		(4)
		Total 6

Question Number	Scheme	Notes	Marks
3(a)	$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + xy = 2y^2$	$y = \frac{1}{z}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{z^2} \frac{\mathrm{d}z}{\mathrm{d}x}$	Correct differentiation	B1
	$-\frac{x^2}{z^2}\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{x}{z} = \frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{\mathrm{d}z}{\mathrm{d}x} - \frac{z}{x} = -\frac{2}{x^2} *$	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
			(3)
(a) Way 2	$y = \frac{1}{z} \Rightarrow zy = 1 \Rightarrow y \frac{dz}{dx} + z \frac{dy}{dx} = 0$	Correct differentiation	B1
	$-\frac{y}{z}x^2\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{x}{z} = -\frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{\mathrm{d}z}{\mathrm{d}x} - \frac{z}{x} = -\frac{2}{x^2} *$	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(a) Way 3	$y = \frac{1}{z} \Rightarrow z = \frac{1}{y} \Rightarrow \frac{dz}{dx} = -\frac{1}{y^2} \frac{dy}{dx}$	Correct differentiation	B1
	$-\frac{1}{y^2} \frac{dy}{dx} - \frac{1}{xy} = -\frac{2}{x^2}$	Substitutes into differential equation (II)	M1
	$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + xy = 2y^2$	Obtains differential equation (I) with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(b)	$I = e^{-\int \frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$	Correct integrating factor of $\frac{1}{x}$	B1
	$\frac{z}{x} = -\int \frac{2}{x^3} \mathrm{d}x$	For $Iz = -\int \frac{2I}{x^2} dx$. Condone the "dx" missing.	M1
	$\frac{z}{x} = \frac{1}{x^2} + c$ $z = \frac{1}{x} + cx$	Correct equation including constant	A1
	$z = \frac{1}{x} + cx$	Correct equation in the required form	A1
			(4)
(c)	$\frac{1}{y} = \frac{1}{x} + cx \Rightarrow -\frac{8}{3} = \frac{1}{3} + 3c \Rightarrow c = -1$	Reverses the substitution and uses the given conditions to find their constant	M1

$\frac{1}{y} = \frac{1}{x} - x \Rightarrow y = \frac{x}{1 - x^2}$	Correct equation for y in terms of x. Allow any correct equivalents e.g. $y = \frac{1}{x^{-1} - x}, y = \frac{1}{\frac{1}{x} - x}$	A1
		(2)
		Total 9

Question Number	Scheme	Notes	Marks
4(a)	$\frac{dy}{dx} = y^2 - x \Longrightarrow \frac{d^2y}{dx^2} = 2y\frac{dy}{dx} - 1$	Correct expression for $\frac{d^2y}{dx^2}$	B1
	$\frac{d^3 y}{dx^3} = 2y \frac{d^2 y}{dx^2} +$ M1 : Applies the product rule to obtain $\frac{d^3 y}{dx^3}$	(41)	M1 A1
	where is no A1 : Correct expression. Ap	on-zero	
	$\frac{d^3 y}{dx^3} = 2y \frac{d^2 y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{d^4 y}{dx^4} = 2$	$dy \frac{d^3y}{dx^3} + 2\frac{dy}{dx}\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\frac{d^2y}{dx^2}$	
	$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = 2y \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} +$	ax ax	A1
	Correct expression for $\frac{d^4y}{dx^4}$ or correct values for A and B.		
	Note:		
	If e.g. $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$ is obtained, allow recovery in (a) so B0M1A1A1 is possible.		
			(4)
(b)	$(y)_{-1} = 1, (y')_{-1} = 2, (y'')_{-1} = 3, (y'')_{-1} = 3$ Attempts the values up to at least the 3rd Condone slips provided the intention is clear.	derivative using $x = -1$ and $y = 1$	M1
	$(y=)1+2(x+1)+\frac{3(x+1)^2}{2}+\frac{14}{2}$ Correct application of the Taylor If the expansion is just written down with no for their values. E.g. $y=-1+\dots$ with no evidence of the second s	series in powers of $(x + 1)$ formula quoted then it must be correct	M1
	$(y=)1+2(x+1)+\frac{3(x+1)^2}{2}+\frac{7}{2}$ Correct simplified expansion. T	$\frac{(x+1)^3}{3} + \frac{8(x+1)^4}{3} + \dots$	A1
			(3)
			Total 7

Question 5 General Guidance

B1: This mark is for sight of -8 seen as part of their working. It may be seen as e.g. embedded in an inequality, as part of their solution if they consider for example x > -8, x < -8 or -8 is seen in a sketch etc.

Do not allow for just e.g. x + 8 > 0,

M1: Any valid attempt to find at least one critical value other than x = -8 (see below).

Condone use of e.g. "=", ">", "<" etc as part of their working.

Note these usually come in pairs as 3, $-\frac{19}{3}$ or 3, -13

M1: A valid attempt to find all critical values.

Condone use of e.g. "=", ">", "<" etc as part of their working.

A1: Any 2 critical values other than x = -8. May be seen embedded in an inequality or on a sketch.

A1: 2 correct regions

A1: All correct with no extra regions

Question Number	Scheme	Notes	Marks
5	(x=)-8	This cv stated or used	B1
	For cv's 3, $-\frac{19}{3}$	OR For cv's 3, -13	
	Examples: $x^2 - 9 = (x+8)(6-2x) \Rightarrow x = \dots$ or	Examples: $x^2 - 9 = -(x+8)(6-2x) \Rightarrow x = \dots$ or	
	$(x^2-9)(x+8) = (x+8)^2(6-2x) \Rightarrow x =$	$-(x^2-9)(x+8) = (x+8)^2(6-2x) \Rightarrow x =$	M1
	$\frac{x^2 - 9}{(x + 8)} - (6 - 2x) = 0 \Rightarrow x =>$ NIP leads to $2x^2 + 10x = 57 = 0$	$\frac{x^2 - 9}{-(x+8)} - (6-2x) = 0 \Rightarrow x =>$ NB leads to $x^2 + 10x - 39 = 0$	
	NB leads to $3x^2 + 10x - 57 = 0$ For cv's 3, $-\frac{19}{3}$	ND For cv's 3, -13	
	Examples: $x^2 - 9 = (x+8)(6-2x) \Rightarrow x = \dots$	Examples: $x^2 - 9 = -(x+8)(6-2x) \Rightarrow x = \dots$	
	$(x^2 - 9)(x + 8) = (x + 8)^2 (6 - 2x) \Rightarrow x = \dots$ or	or $-(x^2-9)(x+8) = (x+8)^2(6-2x) \Rightarrow x =$ or	M1
	$\frac{x^2-9}{(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$	$\frac{x^2-9}{-(x+8)} - (6-2x) = 0 \Rightarrow x = \dots >$	
	NB leads to $3x^2 + 10x - 57 = 0$ Any two of: $x = -13, -\frac{19}{3}, 3$	NB leads to $x^2 + 10x - 39 = 0$ For any two of these cv's. May be seen embedded in their inequalities. Depends	A1
	-13 < x < -8, -8	on at least one previous M mark. $< x < -\frac{19}{3}, x > 3$	
	A1: Any 2 of the Note that $-13 < x < -\frac{19}{3}$, $x \ne -8$ we	-	A1 A1
	Also condone $-13 < x < -\frac{19}{3}$,	x > 3 as 2 correct inequalities.	
	Depends on at least of A1: All correct and no other regions Allow equivalent notation for the ine	s. Depends on all previous marks.	
	x > -13 and $x < -8$, $x > -13$, $x < -8$, $-8 > -8$	$> x > -13, (-13, -8), \{x: x > -13 \cap x < -8\}$	
	But not $x > -1$ Note that $-13 < x < -\frac{19}{3}$, $x < -\frac{19}{3}$		
	3		(6)
			Total 6

Note that it is possible to find all the cv's by squaring both sides of the equation:

(x=)-8	This cv stated or used	B1
$\frac{\left(x^2 - 9\right)^2}{\left(x + 8\right)^2} = 3\left(6 - 2x\right)^2 \Rightarrow x^4 - 18x^2 + 81 = \left(36 - 24x + 4x^2\right)\left(x^2 + 16x + 64\right)$ $\Rightarrow 3x^4 + 40x^3 - 74x^2 - 960x + 2223 = 0 \Rightarrow x = \dots$ M2 Requires a complete attempt to square both sides, multiply up to obtain a quartic equation and an attempt to solve to find at least 1 critical value other than $x = -8$		M1M1
Any two of: $x = -13, -\frac{19}{3}, 3$	For any two of these cv's. May be seen embedded in their inequalities. Depends on both previous M marks.	A1
$-13 < x < -8, -8 < x < -\frac{19}{3}, x > 3$ A1 : Any 2 of these inequalities. Note that $-13 < x < -\frac{19}{3}, x \ne -8$ would count as 2 correct inequalities. Also condone $-13 < x < -\frac{19}{3}, x > 3$ as 2 correct inequalities. Depends on at least one previous M mark.		A1 A1
A1: All correct and no other regions Allow equivalent notation for the ine $x > -13$ and $x < -8$, $x > -13$, $x < -8$, $-8 > -8 > -8 > -8 > -8 > -8 > -8 > -8 $	equalities e.g. for $-13 < x < -8$ allow $> x > -13, (-13, -8), \{x : x > -13 \cap x < -8\}$ -13 or $x < -8$	

Question Number	Scheme	Notes	Marks
6(a)	Im	A straight line anywhere that is not vertical or horizontal which does not pass through the origin. It may be solid or dotted. Clear "V" shapes score M0.	M1
	Re*	A straight line in the correct position. Must have a positive gradient and lie in quadrants 1, 3 and 4. Ignore any intercepts correct or incorrect. If there are other lines that are clearly "construction" lines e.g. a line from 2i to 3 they can be ignored. The line may be solid or dotted. However, if there are clearly several lines then score A0.	A1
			(2)

Part (b)

The approaches below are the ones that have been seen most often. Apply the mark scheme to the overall method the candidate has chosen. There may be several attempts:

- If none are crossed out, mark all attempts and score the best single complete attempt
- If some attempts are crossed out, mark the uncrossed out work
- If everything is crossed out, mark all the work and score the best single complete attempt

Note that the question does not specify the variables the candidates should work in so they may use: e.g. z = x + iy and w = u + iv or w = x + iy and z = u + iv or any other letters so please check the work carefully.

Note that the M marks are all dependent on each other.

(b) Way 1	$w = \frac{iz}{z - 2i} \Longrightarrow z = \frac{2wi}{w - i}$	Attempts to make z the subject. Must obtain the form $\frac{awi}{bw+ci}$, a, b, c real and non-zero.	M1
	$z = \frac{2(u+iv)i}{u+iv-i} \text{ or e.g. } z = \frac{2(x+iy)i}{x+iy-i}$ $z = \frac{2(u+iv)i}{u+(v-1)i} \times \frac{u-(v-1)i}{u-(v-1)i} \text{ or equivalent}$ Introduces $w = u+iv \text{ or e.g. } w = x+iy and attempts to multiply numerator and denominator by the complex conjugate of the denominator. The above statement would be sufficient e.g. no expansion is needed for this mark.$		d M1
	$z = \frac{-2u}{u^2 + (v-1)^2} + \frac{2u^2 + 2v(v-1)}{u^2 + (v-1)^2} i \text{ or e.g.}$ or $z = \frac{-2uv + 2u(v-1) + (2u^2 + 2v(v-1))i}{u^2 + (v-1)^2} \text{ or e.g.}$ Correct expression for z in terms of their v identified. May be embedded as	$z = \frac{-2xy + 2x(y-1) + (2x^2 + 2y(y-1))i}{x^2 + (y-1)^2}$ ariables with real and imaginary parts	A1
	$ z-2i = z-3 \Rightarrow y-1 = \frac{3}{2} \left(x - \frac{3}{2}\right)^2$ $\Rightarrow \frac{2u^2 + 2v(v-1)}{u^2 + (v-1)^2} - 1 = \frac{3}{2}$ Attempts the Cartesian equation of the local equivalent using their variables to obtain an Condone slips with the locus of z but must be a non-zero constant or $ z-2i = z-3 \Rightarrow \left \frac{-2u}{u^2 + (v-1)^2} + \frac{2u^2 + 2v(v-1)}{u^2 + (v-1)^2} \right $ $\Rightarrow \left(\frac{-2u}{u^2 + (v-1)^2}\right)^2 + \left(\frac{2u^2 + 2v(v-1)}{u^2 + (v-1)^2} - 2\right)^2 = \frac{1}{2}$ Substitutes their z into the locus of z and appequation in u and v (or their variables). Note	$\int \left(y = \frac{3}{2}x - \frac{5}{4}, 6x - 4y = 5\right)$ $\frac{3}{2}\left(\frac{-2u}{u^2 + (v - 1)^2} - \frac{3}{2}\right)$ cus of z and substitutes for x and y or equation in u and v (or their variables). We a linear equation in any form but with stant term. $ \mathbf{i} - 2\mathbf{i} = \left \frac{-2u}{u^2 + (v - 1)^2} + \frac{2u^2 + 2v(v - 1)}{u^2 + (v - 1)^2}\mathbf{i} - 3\right $ $\left(\frac{-2u}{u^2 + (v - 1)^2} - 3\right)^2 + \left(\frac{2u^2 + 2v(v - 1)}{u^2 + (v - 1)^2}\right)^2$ Toplies Pythagoras correctly to obtain an	ddM1
	$13u^{2} + 13v^{2} + 12u - 18v + 5 = 0 \Rightarrow v$ $\Rightarrow \left(u + \frac{6}{13}\right)^{2} + \left(v\right)$ Attempts to complete the square on their equation the same coefficient. Award for e.g. $u^{2} + v^{2} + \alpha u + \beta v +$	$\left(\frac{9}{13}\right)^2 = \frac{4}{13}$ quation in u and v where u^2 and v^2 have efficient.	ddd M1

Attempts using the form $u^2 + v^2 + 2gu + 2fv + c = 0$ send to review.		
$\left w - \left(-\frac{6}{13} + \frac{9}{13}i \right) \right = \frac{2}{\sqrt{13}}$	Correct equation in the required form	A1
		Total 8

(b)	Attempts to make z		
Way 2	$w = \frac{iz}{z - 2i} \Rightarrow z = \frac{2wi}{w - i}$ Must obtain the form	$\frac{awi}{bw+ai}$, a, b, c M1	
	z-21 $w-1$ real and non-zero.	DW + C1	
	$ z-2i = z-3 \Rightarrow \left \frac{2wi}{w-i} - 2i \right = \left \frac{2wi}{w-i} - 3 \right $		
	$\Rightarrow \left \frac{2w\mathbf{i} - 2w\mathbf{i} - 2}{w - \mathbf{i}} \right = \left \frac{2w\mathbf{i} - 3w + 3\mathbf{i}}{w - \mathbf{i}} \right $		
	Introduces z in terms of w into the given locus and attempts to combine terms		
	$\left \frac{-2}{w-\mathbf{i}} \right = \left \frac{2w\mathbf{i} - 3w + 3\mathbf{i}}{w-\mathbf{i}} \right \Rightarrow \left -2 \right = \left 2w\mathbf{i} - 3w + 3\mathbf{i} \right $		
	Correct equation with fractions removed	2	
	$ 2(u+iv)i-3(u+iv)+3i = 2 \Rightarrow (3u+2v)^2 + (3v-2u-3)^2 = 4$		
	Introduces e.g. $w = u + iv$ and applies Pythagoras correctly		
	$13u^{2} + 13v^{2} + 12u - 18v + 9 = 4 \Rightarrow u^{2} + v^{2} + \frac{12}{13}u - \frac{18}{13}v + \frac{9}{13} = \frac{4}{13}$		
	$\Rightarrow \left(u + \frac{6}{13}\right)^2 + \left(v - \frac{9}{13}\right)^2 = \frac{4}{13}$		
	Attempts to complete the square on their equation in u and v where u^2 and v^2 have		
	the same coefficient. $(2)^2$		
	Award for e.g. $u^2 + v^2 + \alpha u + \beta v + = \left(u + \frac{\alpha}{2}\right)^2 + \left(v + \frac{\beta}{2}\right)^2 + =$		
	Attempts using the form $u^2 + v^2 + 2gu + 2fv + c = 0$ send to review.		
	$\left w - \left(-\frac{6}{13} + \frac{9}{13}i \right) \right = \frac{2}{\sqrt{13}}$ Correct equation in		
		Tota	18

(b) Way 3	$w = \frac{iz}{z - 2i} \Rightarrow z = \frac{2wi}{w - i}$ Attempts to make z the subject. Must obtain the form $\frac{awi}{bw + ci}$, a, b, c real and non-zero.	M1
	$ z - 2\mathbf{i} = z - 3 \Rightarrow \left \frac{2w\mathbf{i}}{w - \mathbf{i}} - 2\mathbf{i} \right = \left \frac{2w\mathbf{i}}{w - \mathbf{i}} - 3 \right $ $\Rightarrow \left \frac{2w\mathbf{i} - 2w\mathbf{i} - 2}{w - \mathbf{i}} \right = \left \frac{2w\mathbf{i} - 3w + 3\mathbf{i}}{w - \mathbf{i}} \right $	
	$\left \frac{-2}{w - \mathbf{i}} \right = \left \frac{2w\mathbf{i} - 3w + 3\mathbf{i}}{w - \mathbf{i}} \right \Rightarrow \left -2 \right = \left 2w\mathbf{i} - 3w + 3\mathbf{i} \right $	
	Correct equation with fractions removed	
	$ w(2i-3)+3i = (2i-3)(w+\frac{3i}{2i-3}) = 2i-3 w+\frac{6-9i}{13} = 2$	
	Attempts to isolate w and rationalise denominator of other term	
	$\sqrt{13} \left w - \left(-\frac{6}{13} + \frac{9}{13} i \right) \right = 2 \Rightarrow \left w - \left(-\frac{6}{13} + \frac{9}{13} i \right) \right = \frac{2}{\sqrt{13}}$	
	M1: Completes the process by dividing by their $ 2i-3 $	
	A1: Correct equation in the required form	
		(6)

Question Number	Scheme	Notes	Marks
7(a)	Condone use of e.g. $C + iS$ for $\cos x + i \sin x$ if the intention is clear.		
	$(\cos 5x =) \operatorname{Re}(\cos x + i \sin x)^5 = \cos^5 x + \binom{5}{2} \cos^3 x (i \sin x)^2 + \binom{5}{4} \cos x (i \sin x)^4$ Identifies the correct terms of the binomial expansion of $(\cos x + i \sin x)^5$ They may expand $(\cos x + i \sin x)^5$ completely but there must be an attempt to extract the real terms which must have the correct binomial coefficients combined with the correct powers of $\sin x$ and $\cos x$. Condone use of a different variable e.g. θ . $(\cos 5x =) \cos^5 x - 10 \cos^3 x \sin^2 x + 5 \cos x \sin^4 x$ Correct simplified expression. Condone use of a different variable e.g. θ .		M1
			A1
	$\equiv \cos x (\cos^4 x - 10\cos^2 x)$	$x\sin^2 x + 5\sin^4 x$	
	$\equiv \cos x \left(\left(1 - \sin^2 x \right)^2 - 10 \left(1 - \sin^2 x \right)^2 \right)$,	M1
	Applies $\cos^2 x = 1 - \sin^2 x$ to obtain an expression in terms of $\sin x$ inside the bracket. Condone use of a different variable e.g. θ .		
	$\equiv \cos x \left(16\sin^4 x - 12\sin^2 x + 1 \right)$	Correct expression. Must be in terms of x now. The " $\cos 5x$ =" is not required.	A1
			(4)
(b)	$\cos 5\theta = \sin 2\theta \sin \theta - \cos \theta$ $\Rightarrow \cos \theta (16\sin^4 \theta - 12\sin^2 \theta + 1) = 2\sin^2 \theta \cos \theta - \cos \theta$		M1
	$16\sin^4\theta - 14\sin^2\theta$ $\Rightarrow \sin^2\theta = \frac{7 \pm \sqrt{17}}{16}$ Solves for $\sin^2\theta$ by any method including calc least one value for $\sin\theta$. Depends on the first n	$\theta + 2 = 0$ ⇒ $\sin \theta =$ ulator and takes square root to obtain at mark. May be implied by their values of	dM1
	$\sin\theta \text{ or } \theta. \text{ NB } \frac{7\pm\sqrt{17}}{16} = 0.69519, 0.17980$ $\sin\theta = \sqrt{\frac{7\pm\sqrt{17}}{16}} \Rightarrow \theta =$ $\text{NB } \sqrt{\frac{7\pm\sqrt{17}}{16}} = 0.833783, \ 0.424035$ A full method to reach at least one value for θ . Depends on the previous mark. May be implied by their values of θ		ddM1
	$(\theta =) 0.986, 0.438$	Correct values and no others in range. Allow awrt these values.	A1
			Total 8

Note that it is possible to do 7(b) by changing to $\cos \theta$ e.g.

$$\cos\theta \left(16\sin^{4}\theta - 12\sin^{2}\theta + 1\right) = \cos\theta \left(16\left(1 - \cos^{2}\theta\right)^{2} - 12\left(1 - \cos^{2}\theta\right) + 1\right)$$

$$\cos\theta \left(16\left(1 - \cos^{2}\theta\right)^{2} - 12\left(1 - \cos^{2}\theta\right) + 1\right) = 2\sin^{2}\theta\cos\theta - \cos\theta$$

$$16\cos^{4}\theta - 18\cos^{2}\theta + 4 = 0$$

$$\cos^{2}\theta = \frac{9 \pm \sqrt{17}}{16} \Rightarrow \cos\theta = \sqrt{\frac{9 \pm \sqrt{17}}{16}}$$

$$(\theta =)0.986, 0.438$$

This is acceptable as they used part (a) and can be scored as:

M1: Uses part (a) with $\sin^2 \theta = 1 - \cos^2 \theta$ and $\sin 2\theta = 2 \sin \theta \cos \theta$ and collects terms.

dM1: Solves for $\cos^2 \theta$ by any method including calculator and takes square root to obtain at least one value for $\cos \theta$. Depends on the first mark. May be implied by their values of $\cos \theta$ or θ .

NB
$$\frac{9 \pm \sqrt{17}}{16} = 0.82019..., 0.30480...$$

dM1: A full method to reach at least one value for θ . Depends on the previous mark. May be implied by their values of θ

NB
$$\sqrt{\frac{9 \pm \sqrt{17}}{16}} = 0.0.743029..., 0.552092...$$

A1:
$$(\theta =) 0.986, 0.438$$

Question Number	Scheme	Notes	Marks
8(a)	$y = r \sin \theta = (1 - \sin \theta) \sin \theta = \sin \theta - \sin^2 \theta$ $\Rightarrow \frac{dy}{d\theta} = \cos \theta - 2 \sin \theta \cos \theta$ or e.g.	Differentiates $(1-\sin\theta)\sin\theta$ to achieve $\pm\cos\theta\pm k\sin\theta\cos\theta$ or equivalent. Use of $y = r\cos\theta$ or $x = r\cos\theta$ scores M0	M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta - \sin 2\theta$	Correct derivative in any form.	A1
	$\cos \theta - 2\sin \theta \cos \theta = 0 \Rightarrow \cos \theta (1 - 2\sin \theta)$	$(n\theta) = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \dots$	dM1
	Solves to find a value for θ . De		
	$\left(\frac{1}{2},\frac{\pi}{6}\right)$		
	Correct coordinates and no others. Isw if necessary	essary e.g. if written as $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ after	A1
	correct values seen or implied award A1. Allow e.g. $\theta = \frac{\pi}{6}$, $r = \frac{1}{2}$.		
	The value of r must be seen in (a) – i.e	e. do not allow recovery in (b).	(4)
(b) Way 1	Note that the $\frac{1}{2}$ in $\frac{1}{2} \int r^2 d\theta$ is not required for the first 4 marks		
	$\int (1 - \sin \theta)^2 d\theta = \int (1 - 2\sin \theta + \sin^2 \theta) d\theta$	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	M1
	$= \int \left(1 - 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$	$\sin^2\theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$	
	$\int (1-\sin\theta)^2 d\theta = \frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$		
	Correct integration. Condone mixed variables e.g.		A1
	$\int (1-\sin\theta)^2 d\theta = \frac{3}{2}x + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$		
	$\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\frac{1}{2}\right)\left[\left(\frac{\pi}{4} + \sqrt{3} - \frac{\sqrt{3}}{8}\right) - (2)\right]\left(=\frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1\right)$		
	Applies the limits of 0 and their $\frac{\pi}{6}$ to their integration. The $\frac{1}{2}$ is not required.		M1
	For the integration look for at least $\pm \int \sin \theta d\theta \rightarrow \pm \cos \theta$		
	Triangle: $\frac{1}{2} \times \frac{1}{2} \sin \frac{\pi}{6} \times \frac{1}{2} \cos \frac{\pi}{6} \left(= \frac{\sqrt{3}}{32} \right)$		M1
	Uses a correct strategy for the	e area of the triangle	<u> </u>

	Area of $R = \frac{\pi}{8} + \frac{7\sqrt{3}}{16} - 1 + \frac{\sqrt{3}}{32}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32} \left(4\pi + 15\sqrt{3} - 32 \right)$	cao	A1
			(6) Total 10
	Note that the $\frac{1}{2}$ in $\frac{1}{2} \int r^2 d\theta$ is not re-	equired for the first 3 marks	Total 10
(b) Way 2	$\int (1-\sin\theta)^2 d\theta = \int (1-2\sin\theta + \sin^2\theta) d\theta$	Attempts $\left(\frac{1}{2}\right)\int r^2 d\theta$ and applies	M1
	$= \int \left(1 - 2\sin\theta + \frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$	$\sin^2\theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta.$	IVII
	$\int (1-\sin\theta)^2 d\theta = \frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$		
	Correct integration. Condone		A1
	$\int (1-\sin\theta)^2 d\theta = \frac{3}{2}x + 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$		
	$\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta + 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{0}^{\frac{\pi}{2}} = \left(\frac{1}{2}\right)\left[\left(\frac{3\pi}{4} + 0 - 0\right) - (2)\right]\left(=\frac{3\pi}{8} - 1\right)$		
	Evidence of use of both limits 0 and $\frac{\pi}{2}$ to their integration. The $\frac{1}{2}$ is not required.		M1
	For the integration look for at lea	$\operatorname{st} \pm \int \sin \theta \mathrm{d}\theta \to \pm \cos \theta$	
	Triangle – "Segment":		
	$\frac{1}{2} \times \frac{1}{2} \sin \frac{\pi}{6} \times \frac{1}{2} \cos \frac{\pi}{6} - \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \sin \theta)^2 d\theta$		
	$\frac{\sqrt{3}}{32} - \frac{1}{2} \left[\frac{3}{2} \theta + 2 \cos \theta - \frac{1}{4} \sin 2\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(= \frac{15\sqrt{3}}{32} - \frac{\pi}{4} \right)$		M1
	Uses a fully correct strategy for the area above the curve between <i>O</i> and <i>P</i> . Requires a correct method for the triangle as in Way 1 and a correct method for the		
	"segment" using both their $\frac{\pi}{6}$ and $\frac{\pi}{2}$.		
	Area of $R = \frac{3\pi}{8} - 1 + \frac{15\sqrt{3}}{32} - \frac{\pi}{4}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32} \left(4\pi + 15\sqrt{3} - 32 \right)$	cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)(i)	$x = t^{\frac{1}{2}} \Rightarrow \frac{dx}{dy} = \frac{1}{2}t^{-\frac{1}{2}}\frac{dt}{dy} \Rightarrow \frac{dy}{dx} = \dots \text{ or } t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \dots$ Applies the chain rule and proceeds to an expression for $\frac{dy}{dx}$		
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2t^{\frac{1}{2}} \frac{\mathrm{d}y}{\mathrm{d}t}$	Any correct expression for $\frac{dy}{dx}$ in terms of y and t	A1
(a)(ii)	$\frac{dy}{dx} = 2t^{\frac{1}{2}} \frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dt} t^{-\frac{1}{2}} \frac{dt}{dx} + 2t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}$ dM1 : Uses the product rule to differentiate an equation of the form $\frac{dy}{dx} = kt^{\frac{1}{2}} \frac{dy}{dt}$ or		
	equivalent e.g. $\frac{dy}{dx} = kx \frac{dy}{dt}$ to obtain $\frac{d^2y}{dx^2} = \alpha t^{-\frac{1}{2}} \frac{dy}{dt} \frac{dt}{dx} + \dots \text{ or } \frac{d^2y}{dx^2} = \dots + \beta t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}$ or equivalent expressions where is non-zero		
	A1: Any correct expr $\frac{dy}{dt}t^{-\frac{1}{2}}\frac{dt}{dx} + 2t^{\frac{1}{2}}\frac{d^{2}y}{dt^{2}}\frac{dt}{dx} = \frac{dy}{dt}$ $\frac{d^{2}y}{dx^{2}} = 2\frac{dy}{dt} +$ Correct expression in	$\frac{dx}{t^{-\frac{1}{2}} \times 2t^{\frac{1}{2}} + 2t^{\frac{1}{2}} \frac{d^2y}{dt^2} \frac{dt}{dx}}$ $4t \frac{d^2y}{dt^2}$	A1
(b)	$x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - \left(6x^2 + 1\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}} \left(2\frac{\mathrm{d}y}{\mathrm{d}t}\right)$	$+4t\frac{d^{2}y}{dt^{2}}-(6t+1)2t^{\frac{1}{2}}\frac{dy}{dt}+9t^{\frac{3}{2}}y=t^{\frac{5}{2}}$	(5) M1
	Substitutes their expressions from p $2t^{\frac{1}{2}} \frac{dy}{dt} + 4t^{\frac{3}{2}} \frac{d^2y}{dt^2} - 12t^{\frac{3}{2}} \frac{dy}{dt}$ $\Rightarrow 4\frac{d^2y}{dt^2} - 12\frac{dy}{dt}$ Obtains the given answer with no errors and sintermediate line after substitution. Must follow full marks in (a)	$-2t^{\frac{1}{2}}\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$ $-+9y = t^*$ sufficient working shown – at least one attion but check working.	A1*
	Must follow full marks in (a)	apart from SC below.	(2)

Special case in (a) and (b) for those who do not have (a) in terms of y and t only:

$$t = x^{2} \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \dots$$
 Scores M1. ... = $2x \frac{dy}{dt}$ scores A0 in (a)(i)
$$\frac{dy}{dx} = 2x \frac{dy}{dt} \Rightarrow \frac{d^{2}y}{dx^{2}} = 2\frac{dy}{dt} + 2x \frac{d^{2}y}{dt^{2}} \frac{dt}{dx} = 2\frac{dy}{dt} + 4x^{2} \frac{d^{2}y}{dt^{2}}$$
 Scores dM1A1A0 in (a)(ii)
$$x \frac{d^{2}y}{dx^{2}} - (6x^{2} + 1)\frac{dy}{dx} + 9x^{3}y = x^{5} \Rightarrow t^{\frac{1}{2}} \left(2\frac{dy}{dt} + 4x^{2} \frac{d^{2}y}{dt^{2}}\right) - (6t + 1)2x \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$$

$$\Rightarrow t^{\frac{1}{2}} \left(2\frac{dy}{dt} + 4t \frac{d^{2}y}{dt^{2}}\right) - (6t + 1)2t^{\frac{1}{2}} \frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}} \Rightarrow 4\frac{d^{2}y}{dt^{2}} - 12\frac{dy}{dt} + 9y = t^{*}$$
 Scores M1A1 in (b)

Mark (c) and (d) together

(c)	$4m^2 - 12m + 9 = 0 \Rightarrow m = \frac{3}{2}$	Attempts to solve $4m^2 - 12m + 9 = 0$ Apply general guidance for solving a 3TQ if necessary. Correct CF. No need for " $y =$ "	M1
	$(y=)e^{\frac{3}{2}t}(At+B)$	Condone $(y =) e^{\frac{3}{2}x} (Ax + B)$ here but must be in terms of t in the GS. Allow equivalents for the $\frac{3}{2}$.	A1
	$(y =) at + b \Rightarrow \frac{dy}{dt} = a \Rightarrow \frac{d^2 y}{dt^2} = 0$ $\Rightarrow -12a + 9(at + b) = t$		
	Starts with the correct PI form and differentiates to obtain $\frac{dy}{dt} = a$ and $\frac{d^2y}{dt^2} = 0$ and substitutes. NB starting with a PI of $y = at$ is M0		M1
	$9a = 1 \Rightarrow a = \dots$ $9b - 12a = 0 \Rightarrow b = \dots$	Complete method to find a and b by comparing coefficients. Depends on the previous method mark.	dM1
	$y = e^{\frac{3}{2}t} (At + B) + \frac{1}{9}t + \frac{4}{27}$	Correct GS including " $y =$ " and must be in terms of t (no x 's). Allow equivalent exact fractions for the constants.	A1
	3		(5)
(d)	$y = e^{\frac{3}{2}x^2} \left(Ax^2 + B \right) + \frac{1}{9}x^2 + \frac{4}{27}$ Correct equation including " $y = \dots$ " (follow through their answer to (c)). Allow equivalent exact fractions for the constants. For the ft, the answer to (c) must be in terms of t and the answer to (d) should be the same as (c) with t replaced with t and the interms of t if there is no final answer to (c) you can award B1ft if the equation is correct in terms of t if it follows the previous work.		
			B1ft
			(1) Total 13
			1000110